Experimental Results on Incomplete Operational Transition Complexity of Regular Languages

Eva Maia, Nelma Moreira, Rogério Reis

CMUP & DCC, Faculdade de Ciências da Universidade do Porto
Rua do Campo Alegre, 4169-007 Porto, Portugal
e-mail:{emaia,nam,rvr}@dcc.fc.up.pt

1 Regular Languages

We performed some experimental tests in order to analyse how often the upper bounds for state and transition complexity were achieved in practice. Although we fixed the size of the alphabet and consider small values of n and m, the experiments are statistically significant and provide valuable information about the average case behaviour of these operations.

In [1], the authors presented an uniform random generator for complete DFAs. We can use this generator to obtain incomplete DFAs, if we consider the existence of a dead state. However, in this case, the probability that a state has a transition to the dead state is $\frac{1}{n+1}$, where n is the number of useful states of the generated incomplete DFA. Although this corresponds to a uniform distribution, for very large values of n, the referred probability is very low, and thus the generated DFAs are almost always complete. Therefore, in order to generate random incomplete DFAs, we can increase the number of void transitions in the generated DFAs to change the referred probability. For that, the generator accepts a parameter b that defines the multiplicity of dead states. Using b $(0 < b < 1)$, we compute the integer part of $m = \frac{b\times n}{1-b}$, which indicates the number of dead states in generated DFA. Note that the generated DFA becomes more incomplete when b tends to 1.

All the tests were performed using the random generator described above. The tests[1] and the generator[2] were implemented in Python[3], and are both publicly available. In the following experiments we consider $b = 0.7$ (Table 2).

As the DFAs were obtained with a uniform random generator, the size of each sample (20000 elements) is sufficient to ensure a 95% confidence level within a 1% error margin. This is calculated with the formula $n = (\frac{z}{2\epsilon})^2$, where z is obtained from the normal distribution table such that $P(-z < Z < z) = \gamma$, ϵ is the error margin, and γ is the desired confidence level.

The following tables show the results of experimental tests with 20000 pairs of incomplete DFAs as operands. We present the results for operands with $m, n \in \{3, 6, 9, 12, 15, 18, 21, 24, 27\}$ states, such that $m + n = 10$ in Table 1, $m + n = 20$

[1] http://khilas.dcc.fc.up.pt/∼eva/
[2] http://fado.dcc.fc.up.pt
[3] http://www.python.org

in Table 2, and $m + n = 30$ in Table 3, over an alphabet of $k = 5$ symbols. As union and intersection are symmetric operations, we do not need to present all the results. We considered the following measures for the DFA resulting from the operation: the state and transition complexity, sc and tc, respectively; the upper bounds for this measures, $ubsc$ and $ubtc$, respectively; its density $d = \frac{tc}{k*sc}$; and the ratios $rs = \frac{sc}{ubsc}$ and $rt = \frac{tc}{ubtc}$. Note that the results presented on Table 2 are averages, i.e. we calculate all the referred measures for each pair of operands and then we compute the average of each measure. The measures m_1, m_2, m_3 and m_4 are the maximal values of sc, $ubsc$, tc and $ubtc$, respectively. For example, in Table 2, considering $m = 10$ and $n = 10$ we calculate the $ubsc$ for the concatenation of each pair of random incomplete DFAs. Then we do the average of the 20000 obtained values and the result is 8557.90, as we can see in the Table 2. We need to do this because every measure depends of parameters which can be different in each pair of generated DFAs.

b=0.7												
Concatenation												
m	n	sc	$ubsc$	rs	m_1	m_2	tc	$ubtc$	rt	m_3	m_4	d
2	8	18.14	589.28	0.03	134	639	56.85	2920.17	0.019	605	3528	0.58
4	6	17.91	246.11	0.073	112	287	57.23	1192.79	0.05	496	1396	0.603
6	4	16.17	84.91	0.19	54	103	50.89	400.43	0.13	217	470	0.61
8	2	13.24	26.48	0.50	29	33	39.98	116.28	0.344	120	146	0.59
Union												
2	8	14.00	26	0.54	26	26	36.28	73.21	0.50	102	107	0.50
4	6	14.84	34	0.44	33	34	38.98	100.09	0.39	131	144	0.51
Intersection												
2	8	3.45	16	0.22	16	16	3.67	67.33	0.05	37	200	0.13
4	6	3.60	24	0.15	21	24	3.74	104.34	0.04	43	264	0.13
Star												
2		2.06	3.22	0.64	3	4	5.18	8.72	0.59	15	19	0.50
4		4.65	10.70	0.43	12	16	14.04	40.75	0.34	58	74	0.60
6		8.83	38.21	0.23	31	64	30.71	170.84	0.18	136	305	0.68
8		14.34	141.89	0.10	80	256	53.71	676.75	0.08	350	1228	0.73
Reversal												
2		2.43	3	0.81	3	3	5.26	15	0.35	13	15	0.42
4		6.50	15	0.43	15	15	16.65	75	0.22	65	75	0.49
6		12.19	63	0.19	60	63	34.60	315	0.11	261	315	0.54
8		18.67	255	0.07	99	255	55.24	1275	0.04	368	1275	0.56
Complement												
2		2.99	3	1	3	3	9.57	29.71	0.32	15	3	0.64
4		5.00	5	1	5	5	22.14	50.90	0.43	25	5	0.89
6		7	7	1	7	7	33.93	73.87	0.46	35	7	0.97
8		9	9	1	9	9	44.60	95.34	0.47	45	9	0.99

Table 1. Experimental results for general regular languages with $b = 0.7$ and $m + n = 10$.

2 Finite Languages

Similarly to the previous section, we performed some experimental tests in order to analyse the practical behaviour of the operations over finite languages. All the tests were performed with uniformly random generated acyclic DFAs.

b=0.7												
Concatenation												
m	n	sc	$ubsc$	rs	m_1	m_2	tc	$ubtc$	rt	m_3	m_4	d
2	18	54.2	604404.76	0.00009	416	655359	182.90	3141223.07	0.00006	1929	3792832	0.62
4	16	55.85	253077.73	0.0002	430	294911	190.69	1316341.55	0.0001	1962	1533056	0.64
6	14	59.81	88087.17	0.0007	303	106495	210.30	468266.14	0.0004	1377	537856	0.67
8	12	59.11	28115.99	0.002	431	34815	210.50	151521.51	0.001	1928	173280	0.68
10	10	54.79	8557.90	0.01	295	10751	194.56	46208.83	0.004	1378	53300	0.68
12	8	50.72	2523.72	0.02	219	3199	180.17	13481.26	0.01	1001	15568	0.69
14	6	44.73	725.28	0.06	179	927	156.79	3760.56	0.04	750	4336	0.68
16	4	36.35	204.44	0.18	117	263	121.18	1002.60	0.12	481	1171	0.65
18	2	28.16	56.31	0.50	54	71	88.10	250.02	0.35	231	289	0.62
Union												
10	10	33.08	120	0.28	89	120	90.46	378.97	0.24	346	480	0.53
12	8	33.33	116	0.29	89	116	91.87	367.46	0.25	323	463	0.53
14	6	32.38	104	0.31	90	104	88.74	326.77	0.27	336	414	0.53
16	4	29.96	84	0.36	73	84	79.87	255.68	0.31	283	340	0.52
18	2	27.84	56	0.50	55	56	73.23	162.12	0.45	209	225	0.51
Intersection												
10	10	7.98	100	0.08	59	100	9.74	46208.83	0.0002	120	53300	0.19
12	8	8.18	96	0.09	60	96	10.09	445.26	0.02	109	825	0.19
14	6	7.78	84	0.09	56	84	9.58	389.08	0.02	101	722	0.18
16	4	6.61	64	0.10	52	64	7.93	283.61	0.03	99	624	0.17
18	2	6.03	36	0.17	34	36	7.45	155.84	0.05	70	396	0.17
Star												
2		2.07	3.23	0.64	3	4	5.22	8.73	0.60	15	19	0.50
4		4.64	10.72	0.43	12	16	13.96	40.72	0.34	51	74	0.55
6		8.79	38.20	0.23	31	64	30.55	170.63	0.18	136	302	0.68
8		14.39	141.73	0.10	74	256	53.93	676.34	0.08	333	1219	0.73
10		21.61	542.92	0.040	113	1024	85.40	2662.98	0.03	493	4987	0.77
12		30.98	2118.42	0.015	156	4096	127.16	10510.73	0.01	723	19620	0.80
14		41.10	8346.26	0.005	226	12288	173.13	41603.90	0.004	1115	60981	0.83
16		53.20	33113.56	0.002	263	49152	228.74	165364.25	0.001	1209	244731	0.85
18		68.04	131851.28	0.001	304	196608	298.15	658938.51	0.0004	1466	974212	0.87
Reversal												
2		2.43	3	0.81	3	3	5.28	15	0.35	13	15	0.42
4		6.46	15	0.43	15	15	16.48	75	0.22	63	75	0.49
6		12.18	63	0.19	48	63	34.63	315	0.11	181	315	0.54
8		18.72	255	0.07	105	255	55.43	1275	0.043	468	1275	0.56
10		26.46	1023	0.0259	129	1023	80.79	5115	0.0158	536	5115	0.58
12		36.08	4095	0.0088	187	4095	113.74	20475	0.0056	804	20475	0.60
14		45.94	16383	0.0028	224	16383	146.93	81915	0.0018	989	81915	0.61
16		57.05	65535	0.0009	353	65535	185.02	327675	0.0006	1504	327675	0.62
18		70.55	262143	0.0003	337	262143	232.92	1310715	0.0002	1476	1310715	0.63
Complement												
2		3	3	1	3	3	9.62	29.79	0.32	15	55	0.64
4		5	5	1	5	5	22.11	50.81	0.44	25	85	0.88
6		7	8	1	7	7	33.91	73.84	0.46	35	120	0.97
8		9	9	1	9	9	44.61	95.35	0.47	45	155	0.99
10		11	11	1	11	11	54.87	116.90	0.47	55	175	1.00
12		13	13	1	13	13	64.94	140.16	0.46	65	205	1.00
14		15	15	1	15	15	74.99	162.05	0.46	75	235	1.00
16		17	17	1	17	17	85	183.96	0.46	85	265	1.00
18		19	19	1	19	19	95	207.13	0.46	95	280	1.00

Table 2. Experimental results for general regular languages with $b = 0.7$ and $m + n = 20$.

b=0.7												
Concatenation												
m	n	sc	$ubsc$	rs	m_1	m_2	tc	$ubtc$	rt	m_3	m_4	d
3	27	111.58	415729345.15	$2.68E-007$	675	469762047	392.84	2148813548.29	$1.83E-007$	2973	2550000384	0.65
6	24	123.69	90201861.96	$1.37E-006$	833	109051903	446.76	481985249.21	$9.27E-007$	3718	561899520	0.68
9	21	127.87	15965617.18	$8.01E-006$	655	19922943	471.57	87423573.78	$5.39E-006$	2949	99601472	0.71
12	18	123.95	2583546.08	$4.80E-005$	562	3276799	462.06	14316094.85	$3.23E-005$	2586	16376608	0.72
15	15	116.13	396222.28	0.0003	710	507903	436.20	2208805.74	0.0002	3267	2530400	0.73
18	12	103.66	58690.38	0.002	445	73727	388.75	327080.49	0.001	2037	377112	0.73
21	9	88.52	8474.48	0.010	346	10751	327.81	46666.54	0.01	1571	52496	0.72
24	6	71.05	1203.54	0.06	204	1503	254.55	6328.69	0.04	944	7135	0.70
27	3	51.23	167.50	0.31	129	207	170.84	787.64	0.22	615	897	0.65
Union												
3	27	44.35	111	0.40	106	111	119.97	338.77	0.35	405	447	0.52
6	24	49.01	174	0.28	137	174	135.68	554.07	0.24	508	701	0.53
9	21	52.58	219	0.24	158	219	147.90	708.76	0.21	594	869	0.54
12	18	54.33	246	0.22	174	246	153.84	801.55	0.19	643	971	0.55
15	15	55.08	255	0.22	190	255	156.48	832.30	0.19	754	1025	0.55
18	12	54.59	246	0.22	166	246	154.81	802.07	0.19	630	977	0.55
21	9	52.41	219	0.24	159	219	147.29	708.48	0.21	604	872	0.54
24	6	49.05	174	0.28	140	174	135.82	553.96	0.25	521	689	0.53
27	3	44.24	111	0.40	101	111	119.46	338.58	0.35	393	472	0.52
Intersection												
3	27	9.69	81	0.12	70	81	12.73	372.42	0.03	135	803	0.19
6	24	11.47	144	0.08	88	144	14.88	674.86	0.02	160	1254	0.21
9	21	13.11	189	0.07	102	189	17.13	894.09	0.019	196	1620	0.22
12	18	13.77	216	0.064	104	216	17.97	1025.69	0.02	167	1672	0.22
15	15	14.15	225	0.06	132	225	18.55	1068.83	0.02	256	1850	0.22
18	12	13.91	216	0.064	110	216	18.25	1027.45	0.02	189	1728	0.22
21	9	12.96	189	0.07	110	189	16.90	893.04	0.02	193	1568	0.22
24	6	11.47	144	0.08	86	144	14.89	674.49	0.02	168	1220	0.20
27	3	9.62	81	0.12	64	81	12.56	371.72	0.033	139	910	0.19
Star												
3		3.26	5.81	0.56	6	8	9.26	19.56	0.47	29	38	0.56
6		8.84	38.28	0.23	36	64	30.61	171.02	0.18	142	301	0.67
9		17.83	276.68	0.064	112	512	68.85	1343.14	0.05	474	2484	0.75
12		30.93	2117.79	0.01	185	4096	127.14	10506.59	0.01	854	20020	0.80
15		47.58	16615.28	0.003	222	24576	203.27	82915.14	0.002	1009	122339	0.84
18		67.64	131839.72	0.0005	354	196608	296.21	658877.08	0.0004	1721	976641	0.87
21		91.34	1051116.94	$8.69E-005$	558	1310720	407.66	5254933.59	0.00007	2738	6547589	0.88
24		118.94	8397123.8	$1.42E-005$	556	10485760	538.97	41984266.4	$1.28E-005$	2746	52409349	0.90
27		150.11	67138140.7	$2.24E-006$	702	75497472	688.05	335687947.79	$2.05E-006$	3488	377305092	0.91
Reversal												
3		4.36	7	0.62	7	7	10.76	35	0.31	30	35	0.47
6		12.08	63	0.19	44	63	34.2	315	0.11	201	315	0.54
9		22.65	511	0.04	135	511	68.49	2555	0.03	496	2555	0.57
12		36.23	4095	0.01	208	4095	114.2521	20475	0.01	932	20475	0.60
15		52.27	32767	0.002	373	32767	169.45	163835	0.001	1704	163835	0.62
18		70.59	262143	0.0003	396	262143	232.99	1310715	0.0002	1790	1310715	0.63
21		91.43	2097151	$4.36E-005$	764	2097151	306.73	10485755	$2.93E-005$	3452	10485755	0.64
24		114.76	16777215	$6.84E-006$	733	16777215	389.25	83886075	$4.64E-006$	3276	83886075	0.65
27		140.40	134217727	$1.04E-006$	1369	134217727	481.11	671088635	$7.17E-007$	6438	671088635	0.66
Complement												
3		4	4	1	4	4	16.15	41.27	0.39	20	75	0.81
6		7	7	1	7	7	33.93	73.71	0.46	35	120	0.97
9		10	10	1	10	10	49.80	106.80	0.47	50	170	0.99
12		13	13	1	13	13	64.95	140.17	0.46	65	210	1
15		16	16	1	16	16	79.99	173.75	0.46	80	250	1
18		19	19	1	19	19	95	206.95	0.46	95	290	1
21		22	22	1	22	22	110	240.74	0.46	110	340	1
24		25	25	1	25	25	125	274.18	0.46	125	370	1
27		28	28	1	28	28	140	307.99	0.45	140	435	1

Table 3. Experimental results for general regular languages with $b = 0.7$ and $m+n = 30$.

The Tables 4, 5 and 6 show the results of 20000 experimental tests. The number of states of the operands and the measures are the same as used in Section 1.

b=0.7													
Concatenation													
m	n	sc	$ubsc$	rs	m_1	m_2	tc	$ubtc$	rt	m_3	m_4	d
2	8	12.71	21.11	0.60	31	34	47.26	88.52	0.53	136	165	0.72
4	6	15.69	59.45	0.26	40	78	60.23	192.96	0.31	186	383	0.75
6	4	14.56	41.80	0.35	32	43	54.26	149.90	0.36	149	210	0.74
8	2	11.2046	16	0.7002875	16	16	39.39655	65.738	0.5992964495	69	75	0.70
Union												
2	8	8.66	14	0.62	12	14	29.67	49.71	0.60	48	65	0.68
4	6	9.73	22	0.44	16	22	32.30	79.84	0.40	61	103	0.66
Intersection												
2	8	1.77	2	0.89	2	2	1.30	2.33	0.56	5	5	0.13
4	6	3.83	10	0.38	9	10	5.99	14.34	0.42	25	38	0.29
Star												
2		1	0.75	1.34	1	1	2.58	1.93	1.34	5	5	0.52
4		3.05	4.70	0.65	5	7	11.80	15.21	0.78	25	35	0.78
6		7.50	18.82	0.40	23	31	33.17	65.20	0.51	111	153	0.88
8		14.71	71.53	0.21	68	127	68.63	242.52	0.28	332	632	0.93
Reversal												
2		2	2	1	2	2	2.58	2.58	1	5	5	0.26
4		5.57	7.99	0.70	8	8	12.87	31.70	0.41	29	35	0.46
6		11.87	21.00	0.57	20	21	33.98	96.087	0.35	79	100	0.57
8		21.99	62.00	0.35	40	62	70.73	298.14	0.24	160	305	0.64
Complement												
2		3	3	1	3	3	7.74	7.74	1	15	15	0.52
4		5	5	1	5	5	24.13	24.13	1	25	25	0.97
6		7	7	1	7	7	34.97	34.97	1	35	35	1.00
8		9	9	1	9	9	45.00	45.00	1	45	45	1.00

Table 4. Experimental results for finite languages with $m + n = 10$.

References

1. Almeida, M., Moreira, N., Reis, R.: Enumeration and generation with a string automata representation. Theor. Comput. Sci. 387(2), 93–102 (2007)

Concatenation												
m	n	sc	$ubsc$	rs	m_1	m_2	tc	$ubtc$	rt	m_3	m_4	d
2	18	37.11	88.64	0.42	108	159	163.43	417.57	0.39	530	786	0.85
4	16	63.26	634.06	0.1	236	2096	289.27	2913.99	0.10	1147	10471	0.89
6	14	76.09	2480.07	0.03	268	7256	350.93	8249.58	0.04	1305	36267	0.91
8	12	78.28	3803.77	0.02	252	7024	360.60	11050.80	0.03	1236	35105	0.91
10	10	73.52	2670.73	0.03	260	3296	336.23	8314.19	0.04	1285	16463	0.91
12	8	63.8	1158.59	0.06	170	1208	287.97	4143.13	0.07	837	6031	0.90
14	6	51.2	396.78	0.13	123	398	226.61	1615.26	0.14	600	1981	0.88
16	4	37.69	122.99	0.31	75	123	162.18	540.69	0.30	363	610	0.86
18	2	25.09	36	0.70	33	36	104.01	165.38	0.63	152	175	0.83
Union												
10	10	30.95	98	0.32	57	98	125.41	8314.19	0.02	260	16463	0.81
12	8	29.86	94	0.32	52	94	120.75	416.94	0.29	225	455	0.81
14	6	26.55	82	0.32	47	82	106.98	360.17	0.30	203	395	0.80
16	4	21.84	62	0.35	36	62	88.5	267.03	0.33	151	297	0.81
18	2	18.8	34	0.55	22	34	77.06	142.41	0.54	97	163	0.82
Intersection												
10	10	12.93	66	0.20	33	66	29.54	110.7	0.27	106	256	0.44
12	8	11.91	62	0.19	33	62	26.71	102.39	0.26	92	239	0.43
14	6	9.02	50	0.18	25	50	18.96	79.85	0.24	79	168	0.40
16	4	5.02	30	0.17	14	30	8.76	43.92	0.20	39	114	0.33
18	2	1.78	2	0.89	2	2	1.29	2.47	0.52	5	5	0.13
Star												
2		1	0.75	1.33	1	1	2.59	1.94	1.33	5	5	0.52
4		3.05	4.67	0.65	5	7	11.81	15.07	0.78	25	35	0.78
6		7.43	18.82	0.40	23	31	32.86	65.17	0.50	112	154	0.88
8		14.59	71.46	0.20	73	127	68.05	241.79	0.28	362	631	0.93
10		25.11	274.14	0.092	121	511	120.19	888.33	0.135	598	2549	0.955
12		38.75	1066.12	0.036	192	2047	188.05	3297.08	0.057	949	10226	0.969
14		57.18	4190.58	0.014	416	8191	279.82	12436.48	0.023	2078	40896	0.977
16		79.35	16599.54	0.005	481	32767	390.42	47644.04	0.008	2400	163810	0.982
18		108.37	66019.6	0.002	751	98303	535.28	184747.27	0.003	3745	491492	0.986
Reversal												
2		2	2	1	2	2	2.59	2.59	1	5	5	0.26
4		5.58	7.99	0.70	8	8	12.93	31.72	0.41	29	35	0.46
6		11.87	20.10	0.57	20	21	33.96	96.08	0.35	76	100	0.57
8		21.99	62.00	0.35	44	62	70.66	298.14	0.24	182	305	0.63
10		37.35	158	0.24	94	158	129.88	779.05	0.17	401	785	0.69
12		59.34	411	0.14	144	411	217.31	2042.01	0.11	640	2050	0.72
14		89.91	1179	0.08	247	1179	342.91	5882.71	0.06	1115	5890	0.75
16		130.19	2828	0.05	355	2828	511.46	14126.25	0.04	1629	14135	0.78
18		184.32	8001	0.02	460	8001	742.56	39989.92	0.02	2057	40000	0.80
Complement												
2		3	3	1	3	3	7.77	8	1	8	8	1
4		5	5	1	5	5	24.12	25	1	25	25	1
6		7	7	1	7	7	34.97	35	1	35	35	1
8		9	9	1	9	9	45	45	1	45	45	1
10		11	11	1	11	11	55	55	1	55	55	1
12		13	13	1	13	13	65	65	1	65	$6s5$	1
14		15	15	1	15	15	75	75	1	75	75	1
16		17	17	1	17	17	85	85	1	85	85	1
18		19	19	1	19	19	95	95	1	95	95	1

Table 5. Experimental results for finite languages with $m + n = 20$.

m	n	sc	$ubsc$	rs	m_1	m_2	tc	$ubtc$	rt	m_3	m_4	d
b=0.7												
Concatenation												
3	27	107.88	935.81295	0.12	460	2982	509.35	4629.75	0.11	2238	14897	0.92
6	24	190.92	18397.83	0.01	732	149404	918.13	86082.33	0.01	3634	747003	0.95
9	21	228.89	136133.61	0.002	983	793515	1104.54	415060.44	0.003	4865	3967556	0.96
12	18	232.11	230010.06	0.001	800	574960	1119.32	230010.06	0.005	3978	574960	0.96
15	15	209.99	110982.76	0.0019	651	145825	1008.01	304674.4	0.003	3191	729085	0.96
18	12	173.74	26472.94	0.007	507	27491	827.64	89009.11	0.009	2483	137432	0.95
21	9	131.18	4583.331	0.03	363	4591	617.36	18381.13	0.03	1794	22941	0.94
24	6	87.97	717.99	0.12	229	718	405.25	3194.60	0.13	1112	3583	0.92
27	3	49.31	107	0.46	83	107	219.4	501.65	0.44	380	530	0.89
Union												
3	27	30.11	79	0.38	39	79	130.673	352.80	0.370	185	382	0.87
6	24	40.42	142	0.28	67	142	173.68	652.51	0.27	300	694	0.86
9	21	50.89	187	0.27	90	187	220.07	870.82	0.25	427	915	0.86
12	18	57.72	214	0.27	100	214	251.39	1002.65	0.25	470	1055	0.87
15	15	59.98	223	0.27	106	223	261.71	1046.67	0.25	479	1097	0.87
Intersection												
3	27	3.22	27	0.12	7	27	4.41	35.56	0.12	20	104	0.25
6	24	10.99	90	0.12	33	90	24.03	147.55	0.16	110	314	0.42
9	21	19.48	135	0.14	57	135	47.67	238.13	0.20	174	468	0.47
12	18	25.46	162	0.16	64	162	65.21	294.79	0.22	212	549	0.5
15	15	27.42	171	0.16	68	171	70.99	313.94	0.23	220	563	0.5
Star												
3		1.79	2.12	0.84	2	3	6.03	6.16	0.98	10	15	0.69
6		7.45	18.77	0.4	23	31	32.98	64.93	0.51	112	154	0.88
9		19.57	140.00	0.14	112	255	92.6	466.37	0.2	556	1271	0.94
12		38.76	1065.84	0.04	180	2047	188.03	3292.71	0.06	892	10223	0.97
15		67.49	8336.19	0.01	552	16383	331.27	24331.3	0.014	2746	81893	0.98
18		108.73	66032.57	0.001	768	98303	537.09	184292.67	0.003	3832	491485	0.99
21		162.17	525961.94	0.0003	951	655359	803.93	1418262.95	0.0005	4749	3276764	0.99
24		230.41	4199704.45	$5.49E-005$	1538	4718591	1144.92	10993007.99	0.0001	7680	23592931	0.99
27		318.77	33573422.57	$9.49E-006$	3592	37748735	1586.49	85724782.36	$1.85E-005$	17932	173015028	0.99
Reversal												
3		3.50	4	0.88	4	4	6.60	11.17	0.59	13	15	0.38
6		11.87	21	0.57	19	21	33.99	96.13	0.35	74	100	0.57
9		28.93	94	0.31	57	94	97	457.28	0.21	229	465	0.66
12		59.11	411	0.14	133	411	216.247	2041.93	0.11	555	2050	0.72
15		108.54	1804	0.06	298	1804	420.16	9004.50	0.047	1293	9015	0.77
18		184.22	8001	0.02	469	8001	741.79	39989.89	0.02	2187	40000	0.80
21		294.14	35914	0.01	829	35914	1218.61	179549.38	0.01	3824	179565	0.82
24		449.71	150602	0.002	1266	150602	1905.46	752994.38	0.003	5954	753005	0.84
27		661.17	621943	0.001	1955	621943	2849.88	3109693.80	0.0009	9265	3109710	0.85
Complement												
3		4	4	1	4	4	17.013	17.013	1	20	20	0.85
6		7	7	1	7	7	34.97	34.97	1	35	35	1
9		10	10	1	10	10	50	50	1	50	50	1
12		13	13	1	13	13	65	65	1	65	65	1
15		16	16	1	16	16	80	80	1	80	80	1
18		19	19	1	19	19	95	95	1	95	95	1
21		22	22	1	22	22	110	110	1	110	110	1
24		25	25	1	25	25	125	125	1	125	125	1
27		28	28	1	28	28	140	140	1	140	140	1

Table 6. Experimental results for finite languages with $m+n=30$.